Chapter 7: Techniques of Integration

Section 7.1: Integration by Parts

Objective: In this lesson, you learn

 \square how to obtain integration by parts as the rule corresponding to the Product Rule for differentiation and apply it to evaluate integrals.

I. Integration by Parts

Problem: Integrate

What is the antiderivative of
$$xe^{x}$$
?

1. let $u = \times \rightarrow du = d \times$

Every differentiation rule has a corresponding integration rule. For example, the **Substitution** Rule for integration corresponds to the Chain Rule for differentiation.

The rule that corresponds to the **Product Rule for differentiation** is called the rule for **integration by parts.**

Recall that the Product Rule states that if f(x) and g(x) are differentiable functions, then

$$\int \frac{d}{dx} \left[f(x)g(x) \right] = \int f(x)g'(x) + \int g(x)f'(x) dx$$

In the notation for indefinite integrals, this equation becomes

$$\int \left[f(x)g'(x) + g(x)f'(x) \right] dx = f(x)g(x)$$

or

$$\int f(x)g'(x)dx + \int g(x)f'(x)dx = f(x)g(x)$$

Rearrange the equation to write the formula for **integration by parts**:

$$\int \underbrace{f(x)g'(x)dx}_{\text{QV}} = \underbrace{f(x)g(x)}_{\text{W}} - \int \underbrace{g(x)f'(x)dx}_{\text{QV}}$$

In order to remember the formula more easily, let

$$u = f(x)$$
 \Longrightarrow $du = f'(x)dx,$ $dv = g'(x)dx$ \Longrightarrow $v = g(x)$

So the formula above becomes:

$$\int u dv = uv - \int v du$$

Note that the purpose of using this formula is to obtain a simpler integral so choose u and vcarefully so that the resulting integral is simpler than the original one.

Example 1: Integrate

Example 1: Integrate

$$\int xe^{x} dx$$

$$u = x \qquad du = dx$$

$$dv = e^{x} dx \qquad f = e^{x}$$

$$\int u dv = uv - \int v du$$

$$\int xe^{x} dx = xe^{x} - e^{x} dx$$

Choose u and v carefully!

Choose a u that gets simpler when you differentiate it and a v that doesn't get any more complicated when you integrate it.

A helpful rule of thumb is I LATE, choose u based on which of these comes first:

- I: Inverse trigonometric functions such as $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$.
- L: Logarithmic functions such as ln(x), log(x).
- A: Algebraic functions such as x^2, x^3 .
- T: Trigonometric functions such as $\sin(x)$, $\cos(x)$, $\tan(x)$
- E: Exponential functions such as e^x , 3^x .

Example 2: Integrate
$$\int x^{2}e^{x}dx$$

$$u = x^{2} \qquad D \qquad du = 2x dx$$

$$v = e^{x}dx \qquad T \qquad V = e^{x}$$

$$\int x^{2}e^{x}dx = x^{2}e^{x} - \int 2x e^{x}dx$$

$$= x^{2}e^{x} - 2 \int xe^{x}dx \qquad From Example 1$$

$$= x^{2}e^{x} - 2 \cdot (xe^{x} - e^{x}) + C$$

$$= x^{2}e^{x} - 2 \cdot (xe^{x} + 2e^{x} + C)$$

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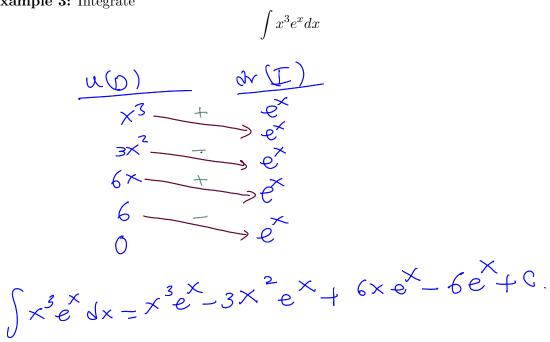
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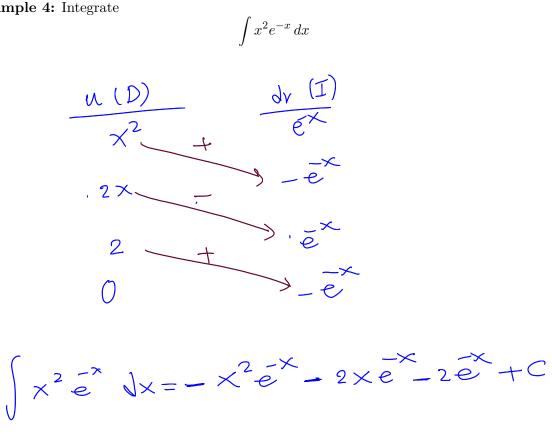
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Example 3: Integrate



Example 4: Integrate



Example 5: Integrate

$$\int x \cos x dx$$

$$U(D) \qquad dv(I)$$

$$\times \qquad + \qquad \cos x$$

$$-\sin x$$

$$0 \qquad -\cos x$$

$$\int \times \cos x \, dx = - \times \sin x + \cos x + C$$

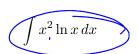
Example 6: Integrate

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$$\int \ln x \, dx$$

$$\int \ln$$

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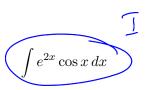
Example 7: Integrate



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Example 8: Integrate

Example 9: Integrate





$$dv = e^{2x} dx \longrightarrow r = \frac{1}{2} e^{2x}$$

$$\overline{J} = \frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x \, dx$$

$$u = 630 \times \frac{D}{\sqrt{1 - \sqrt{4}}} du = \cos x \, dx$$

$$dv = e^{2x} \times \frac{D}{\sqrt{1 - \sqrt{4}}} e^{2x}$$

$$T$$

$$I = \frac{1}{2} e^{2x} \cos x + \frac{1}{2} \left[\frac{1}{2} e^{2x} \sin x - \frac{1}{2} \left(\frac{e^{2x} \cos x}{e^{2x}} dx \right) \right]$$

$$I = \frac{1}{2}e^{2x}\cos x + \frac{1}{4}e^{2x}\sin x - \frac{1}{4}I$$

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$$I = \frac{4}{9} \left(\frac{1}{2} e^{2x} \cos x + \frac{1}{4} e^{2x} \sin x \right) + C$$

Example 10: Integrate

Example 10: Integrate
$$\int x \tan^{2}(x) dx$$

$$\int x \cot^{2}(x) dx$$

$$\int x \cot^$$

 $\int x \tan^2(x) \, dx$

 $= \times + an \times - x^2 - Ln|secx| + \frac{x^2}{2} + C$

Theorem

$$\int_{a}^{b} f(x)g'(x) dx = \underline{f(x)}\underline{g(x)}\Big|_{a}^{b} - \int_{a}^{b} g(x)f'(x) dx.$$

Example 12: Evaluate